

Test 2A - MTH 1420
Dr. Graham-Squire, Spring 2013

Name: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(signature)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones are not allowed on this test. Computers and/or calculators are allowed on the first question. Calculators (but not computers) are allowed on all other questions, however you should still show all of your work to receive full credit. If you are asked to integrate something, I expect you to integrate it by hand unless otherwise specified.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. If you have a problem involving trigonometric substitution or integration, these may help:
 - If we have a factor of the form $\sqrt{a^2 - x^2}$, we do the substitution $x = a \sin \theta$.
 - If we have a factor of the form $\sqrt{a^2 + x^2}$, we do the substitution $x = a \tan \theta$.
 - If we have a factor of the form $\sqrt{x^2 - a^2}$, we do the substitution $x = a \sec \theta$.
 - $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$
 - $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$
6. Make sure you sign the pledge.
7. Number of questions = 8. Total Points = 80.

Calculator and Computers are okay

1. (10 points) Find the arc length of the function $y = e^x \sin x$ on the interval $2 \leq x \leq 6$. Simplify the integral as much as possible, and if you cannot evaluate the integral by hand, use a calculator or Sage/Maple to do numerical integration. Round your answer to the nearest 0.01.

Calculators are okay, no computers allowed

2. (10 points) Three integrals are given below. For only one of them is it necessary to do trigonometric substitution.

$$\int_{3/2}^3 \frac{x}{\sqrt{x^2 - 9}} dx$$

$$\int_{3/2}^3 \frac{x^2 + 9}{x^2} dx$$

$$\int_{3/2}^3 \sqrt{9 - x^2} dx$$

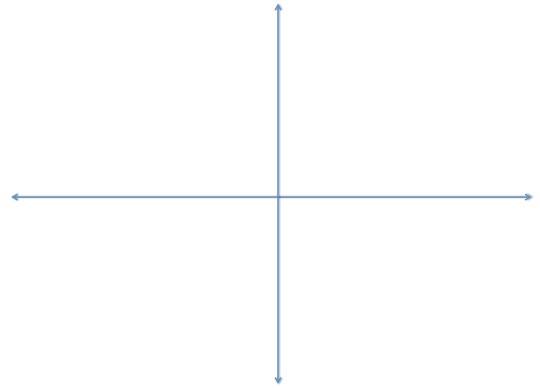
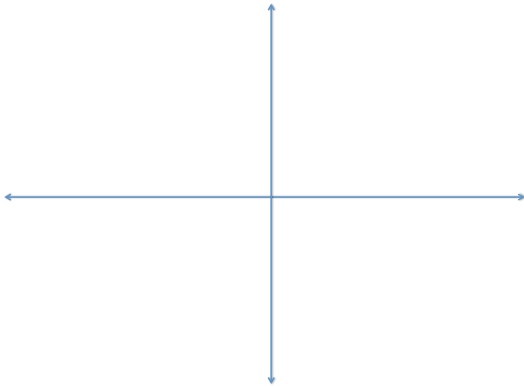
(a) Which of the two do not require trigonometric substitution? For each, explain how you would integrate it (i.e., what integration method you would use to solve it), but **do not actually do the integration**.

(b) Use trigonometric substitution to integrate the one that needs trigonometric substitution.

3. (10 points) Below, two solids of revolution are described. You should choose one of them and set up the integral to represent how to find the volume of the solid. **You do not need to set up both integrals, only one is necessary.** If you set up both, you must circle which one you want me to grade.

(i) The region R in the first quadrant enclosed by the curve $y = -x^4 + 3x^3$ and the x -axis, rotated about the y -axis.

(ii) The region S in the first quadrant enclosed by the curves $y^2 = x$ and $y = \frac{x^2}{8}$, rotated about the x -axis.



4. (10 points) Let S be the region in the first quadrant bounded by the lines $x = 0$, $y = 8$, and $y = x^3$. Set up, but **do not integrate** an integral that represents the volume of the solid with base S such that cross-sections perpendicular to the x -axis are isosceles right triangles with one leg (not the hypotenuse) lying on S .



5. (10 points) Are the following integrals improper or not? For each improper integral, express it using the appropriate limit notation. If the integral is *not* improper, just say 'not improper'. **Do not actually integrate any of these!**

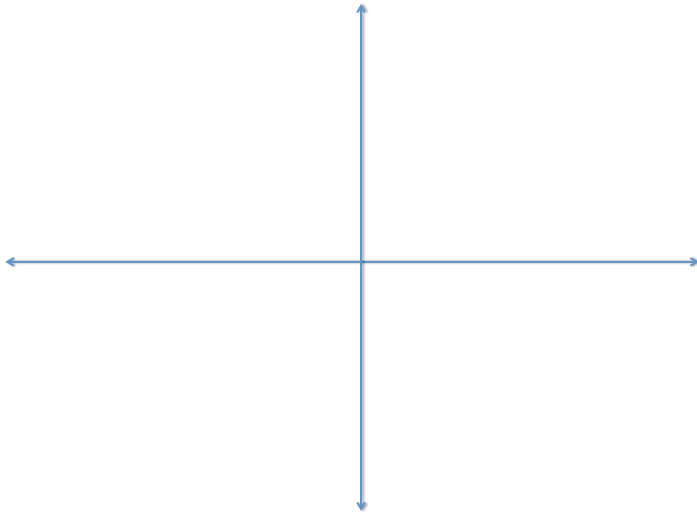
(i) $\int_0^{3\pi/4} \tan x \, dx$

(ii) $\int_2^7 \sin(\ln x) \, dx$

(iii) $\int_0^3 \frac{x-1}{x^2-6x+9} \, dx$

(iv) $\int_0^4 \frac{x-4}{e^x} \, dx$

6. (10 points) Find the area bounded by $y = x \ln x$, $y = 2x$, and the line $x = 1$. Leave your answer in exact form.



7. (10 points) A tank filled with water is in the shape of an inverted cone, with the pointy end down. The base of the cone has a radius of 3 feet, and the height of the cone is 9 feet. Set up, but **do not integrate**, an integral to find the amount of work needed to pump all of the water out of the top of the tank. Assume that water has a density of 62.5 lb/ft^3 .

8. (10 points) Is the integral

$$\int_3^{\infty} \frac{1}{(x-1)^3} dx$$

convergent or divergent? If convergent, what is the value of the integral? Make sure to use correct mathematical notation.

Extra Credit(2 points) Either graph and use geometry, or use the arc length formula, to find the arc length of the parametric curve $x = 4 \cos \theta$, $y = 4 \sin \theta$ for $0 \leq \theta \leq \pi$.